# CTBT Network Design Sensitivity


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## Introduction

It is possible to make some simple calculations to estimate the ability of a sampling network to detect atmospheric contaminants without reference to a particular meteorological model or specific network design. One can calculate the downwind concentration of a pollutant and estimate its horizontal extent. If we assume that the probability of the travel path is uniform over all compass directions, then the probability of detection at any downwind distance is simply the ratio of the plume width over the sampler spacing. A plume extent that exceeds the sampler spacing would have a 100% detection probability. For a narrow plume, assume one that is only half as wide as the sampler spacing, then the probability of detection will be 50%, because in only half the downwind directions could a plume intersect a sampling location.

## Calculation

The calculation procedure is very simple and can be adapted to most spreadsheet programs or other computational methods (see Appendix). The centerline concentration

$$\chi(0) = 2\, Q \,/\, [\, (2\, \pi)^{3/2}\ \sigma_x\, \sigma_y\, \sigma_z\ ],$$

where Q is the emission amount, and $\sigma_x$ , $\sigma_y$, and $\sigma_z$ are the horizontal and vertical dispersion parameters. For simplicity one can assume that

$$\sigma_x = \sigma_y = \sigma_h = 0.5\ t,$$

where t is in seconds and $\sigma_h$ is in meters. The vertical dispersion parameter can be estimated from an average tropospheric diffusivity such that

$$\sigma_z = \sqrt{2\, K_z\, t},$$

where $K_z$ is assumed to equal 5 $m^2/s$. The concentration normal to the centerline is estimated from a Gaussian distribution

$$\chi(y) = \chi(0) \exp( -0.5 ( y / \sigma_y )2 ),$$

where y is the distance from the centerline. Concentrations are further reduced by radioactive decay (or other removal processes) through the relation

$$e^{t\beta},$$

where $\beta$ is the decay time constant equal to ln 0.5 divided by the pollutant's half-life. The plume extent, or width (W) , is calculated as the minimum value of $\pm 3\ \sigma_y$ or the distance from edge to edge at which the plume concentration falls below the sampler detection limit. The $\pm 3\ \sigma_y$ limits encompass 99% of the mass in a Gaussian distribution.

The detection probability is computed from the sampler spacing. The distance between samplers is computed from the assumption that the designated number of samplers (N) are uniformly distributed over the surface of the earth and therefore the average areal coverage of each sampler

$$A = (4 \pi r^2) / N,$$

where the radius (r) of the earth equals 6371.2 km. The spacing (S) is then the diameter of a circle with the area A,

$$S = 2 \sqrt{\ } A / \pi ,$$

and the detection probability

$$P = 100 W / S.$$

## Results

All calculations are performed for Ba-140 with a half-life of 12.75 days, an emission amount (Q) of $2.2 \times 10^{15}$ Bq, and a sampler detection limit of $1 \times 10^{-6}$ Bq/scm. Only the 60- and 100-network station configuration will be evaluated in this paper. The results from the 60 station calculations are summarized in Table 1 and show higher probabilities than those of Rodhe and Hamrud (1985). They reported a detection probability of 41% after 10 days versus the 79% shown in Table 1. However, when they doubled their horizontal dispersion, their results more closely matched those reported in this study. Rodhe and Hamrud used a horizontal dispersion equation for the plume radius of 600 $\sqrt{t}$, which is substantially smaller than the horizontal growth rate assumed in these calculations. Their equation would give larger values for the first 2-3 days travel. However, after 10 days the horizontal dispersion used in these calculations gives plume sizes that are more than twice as large as those of Rodhe and Hamrud. There is considerable empirical evidence to support the linear relation 0.5 t (see Gifford, 1977) and it has been repeatedly demonstrated that this equation passes through the bulk of the data observations. However one can argue that these experimental data represent a biased sample in that observational data are not usually taken during synoptic events that create large plume distortions (Gifford, 1989) and that if the experimental data base were to represent all the potential synoptic events, then a true climatological horizontal dispersion coefficient would be even larger.

Table 1. Detection probability for the 60 station network using standard model parameters.

| Days | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Probability | 8 | 16 | 24 | 32 | 39 | 47 | 55 | 63 | 71 | 79 |

Extending the Gaussian model to the 100 station network (Table 2) provides an opportunity to compare the estimates with the 100 station results reported by Mason (1995). He found that detection probabilities after 5 days were near 80% and above 90% after 10 days. Mason used a meteorological model that included the additional dispersive effects of horizontal wind shear and large scale synoptic disturbances that typically would tear apart a Gaussian puff. Probabilities using the Gaussian only exceeded 50% after 5 days and were very similar to Mason's results by 10 days travel. The differences at the shorter travel times are understandable because Mason's 100 station network has some spatial variation that in conjunction with sampler orientation to specific source locations, can have a considerable effect on detection probabilities. Considering the nature of the model presented here, the results shown in Table 2 are very encouraging, especially at the longer travel times.

Table 2. Detection probability for the 100 station network using standard model parameters.

| Days | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Probability | 10 | 20 | 31 | 41 | 51 | 61 | 71 | 81 | 92 | 100 |

A simple estimate for the synoptic influence on horizontal dispersion was given by Draxler and Taylor (1982). A theoretical study following the dispersion of a "puff" within the boundary layer, where the winds turn following an Ekman profile, showed that the horizontal dispersion was 50% or greater than suggested by the 0.5 t relationship. Other studies summarized in that paper showed horizontal dispersion estimates could be higher by a factor of 2-3 when pollutants were initially distributed over deeper vertical layers. These effects can simply be included in the Gaussian model by just increasing the horizontal growth by a factor of two over the base case to demonstrate detection sensitivity to plume size.

The 2x enhanced horizontal dispersion (Table 3) doubled the detection probabilities over those shown in Table 2. The detection probabilities for the first few days are almost identical to those reported by Mason, who explicitly includes the effect of more realistic enhanced horizontal dispersion computations by using actual meteorological data within the calculations. However the Gaussian results do show detection probabilities approaching 100% much earlier that in Mason's study.

Table 3. Detection probability for the 100 station network using a factor of two increased horizontal dispersion.

| Days | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Probability | 20 | 41 | 61 | 81 | 100 | 100 | 100 | 100 | 100 | 100 |

## Conclusions

A simple Gaussian model, independent of any meteorological assumptions or network configuration design, was used to confirm the results from more detailed and realistic model calculations. The detection probabilities for a 100-station network as reported by Mason (1995) are consistent with our current understanding of atmospheric dispersion processes. Based on the calculations presented here and those of previous studies, one can conclude that detection probabilities for Ba-140 are in the range of 50% - 80% by 5 days travel time, depending upon the network spatial configuration and meteorological conditions. Detection probabilities approach 90% at 10 days travel regardless of any meteorological, dispersion, or design considerations.

## References

Draxler, R.R, and A.D. Taylor, 1982, Horizontal dispersion parameters for long-range transport modeling, *J. Appl. Meteorol., 21*, 367-372.

Gifford, F.A., 1977, Tropospheric relative diffusion observations, *J. Appl. Meteorol., 16*, 311-313.

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Mason, L.R., 1995, Comprehensive design analysis for an international radionuclide monitoring system, PSR Corp, 96p.

Rodhe, H., and M. Hamrud, 1985, On the design of a global detection system for airborne radioactivity, Arrhenius Laboratory, 1985-01-15, 24p.

## Appendix - Model Fortran Source Code

```
C sampler density
        nsam=100
        write(*,*)'Enter number of samplers-'
        read(*,*)nsam
C scan radius in horizontal sigma units
        scan=3.0
        write(*,*)'Enter horizontal scan radius-'
        read(*,*)scan
C horizontal diffusion multiplier
        hmult=1.0
        write(*,*)'Enter horizontal diffusion multiplyer:'
        read(*,*)hmult
C vertical diffusion coefficient (m^2/s)
        vmix=5.0
C source term in Bq
        qval=2.2e+15
C sampler detection limit in Bq/scm
        samp=1.0e-06
C half-life in days
        half=12.75
        decay=alog(0.5)/(half*86400.0)
C circle constant
        pi=3.14159
C dispersion equation constant
        const=0.5*(2.0*pi)**(3.0/2.0)
C earth's radius (m)
        radius=6371200.0
C earth's surface area (m^2)
        earth=4.0*pi*radius*radius
C sampler area coverage (m^2)
        area=earth/nsam
C sampler distance spacing (m)
        sdist=2.0*sqrt(area/pi)

        write(*,'(10a10)')'time','sig-y','sig-z','max-con','detection',
   :      'plume','percent'

C loop through time in hours
        do days=1,10
                hours=days*24.0

C               horizontal standard deviation (m)
                hsigma=1800.0*hours*hmult
C               vertical standard deviation (m)
                vsigma=sqrt(2.0*vmix*hours*3600.0)
C               center-line concentration
                conc0=1.0/(const*hsigma*hsigma*vsigma)
C               apply decay term
                conc0=conc0*exp(hours*3600.0*decay)
C               distance at which concentration below detection
                 ydist=hsigma*sqrt(2.0*alog(samp/conc0))
C               plume width distance
                plume=2.0*min(ydist,scan*hsigma)
```

```
C                detection probability
                 prob=min(100.0,100.0*plume/sdist)

                 write(*,'(3f10.0,e10.2,3f10.0)')
     :            days,hsigma,vsigma,conc0,ydist,plume,prob
          end do
          end
```